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Hot Carrier Effects in the Integral Charge-Control Model for Bipolar Transistors

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The integral charge-control model for bipolar transistors is rederived with the purpose of elucidating hot carrier effects. In its original derivation the model contained an additive hot carrier contribution to the base charge of possible significance in narrow-base transistors. Inclusion of this term is shown to be unnecessary. However, careful examination of the potentials appearing in the formalism has disclosed other hot carrier effects. These could lower the transconductance of a transistor operating in or near saturation, particularly if the base has a low number of impurities per unit area, but would otherwise be unobservable.

I. INTRODUCTION

The integral charge-control model (ICM) provides an elegant and compact description of the one-dimensional transport physics of transistors by relating collector current to the junction voltages and total base majority carrier charge.[1,2] The original derivation of the model indicates a possible need for supplementing the base charge in the ICM relation with a term inversely proportional to the minority carrier saturation velocity when base widths are very small (~1000 Å).[1] It is shown herein that this term is an artifact arising from inappropriate treatment of the diffusion current contribution to the transport equation. There are, however, additional hot carrier modifications of the charge-control relation that have not been included in previous treatments. These originate in the heating of a reverse current by the built-in field in a junction *not* supporting a large reverse bias, and should be manifest only in saturated or near-saturated transistor operation. With this exception, the standard ICM relation [equation (15) of Ref. 1] remains valid to the same extent as the macroscopic current transport equation, even for very narrow base widths.

II. DERIVATION

Considering a pnp transistor, we integrate the one-dimensional macroscopic equation for hole transport to obtain the integral charge-control relation. Our derivation largely parallels that of H. K. Gummel.[1] The essential differences are representation of diffusive transport by $-q\nabla(Dp)$ rather than $-qD\nabla p$, and a more detailed treatment of the potentials. Both diffusion expressions are, of course, identical if D is coordinate independent. When coordinate dependencies arise from local carrier heating, the former can be more readily justified by integration of the Boltzmann equation, and is therefore to be preferred.[3] Thus, as a starting equation we take

$$j_p = q\left(\frac{qE}{kT_d}\eta - \frac{d\eta}{dx}\right), \tag{1}$$

where j_p is the hole current density, E is the electric field, and kT_d and η are given by

$$kT_d \equiv qD/\mu, \tag{2}$$

$$\eta \equiv Dp. \tag{3}$$

In relation (2), T_d is the hole "diffusion temperature," which is *defined* from the local diffusion coefficient and mobility by the Einstein relation. The variable η is the product of the local diffusion coefficient and hole density.

The full solution to (1) is the sum of the homogeneous solution for $j_p = 0$, and the particular solution. From the homogeneous equation we obtain

$$\eta_h = e^{-\psi(x)}, \tag{4}$$

where

$$\frac{d\psi}{dx} = -\frac{qE}{kT_d}. \tag{5}$$

Note that ψ is a potential normalized to the local value of kT_d, and is nonconservative in regions where T_d varies. The particular solution to (1) is

$$\eta_p = -\frac{1}{q}e^{-\psi(x)}\int^x j_p(x')e^{\psi(x')}\,dx'. \tag{6}$$

Thus

$$\eta = e^{-\psi(x)} - \frac{1}{q}e^{-\psi(x)}\int^x j_p(x')e^{\psi(x')}\,dx'. \tag{7}$$

Equation (7) is now evaluated at x_E, the outer edge of the emitter junction, and x_C, the outer edge of the collector junction. This procedure yields

$$\eta(x_E)e^{\psi(x_E)} - \eta(x_C)e^{\psi(x_C)} = \frac{1}{q}\int_{x_E}^{x_C} j_p(x')e^{\psi(x')}\,dx'. \tag{8}$$

Following Gummel,[1] we may account in a crude way for recombination through the introduction of a quantity $\bar{a}$ defined by

$$\int_{x_E}^{x_C} j_p(x)e^{\psi(x)}\,dx = \bar{a}j_c\int_{x_E}^{x_C} e^{\psi(x)}\,dx, \tag{9}$$

where $j_c = j_p(x_C)$ is the collector hole current density. Consequently, $\bar{a} \geqq 1$, and assumes the value unity in the absence of recombination. Upon substitution of (9) into (8), the resulting equation may be solved for j_c.

$$j_c = \frac{q}{\bar{a}}\frac{\eta(x_E)e^{\psi(x_E)} - \eta(x_C)e^{\psi(x_C)}}{\displaystyle\int_{x_E}^{x_C} e^{\psi(x)}\,dx}. \tag{10}$$

There remains evaluation of the contributions to (10). At coordinates x_E and x_C in the undepleted bulk material of the emitter and collector there is no carrier heating and the diffusion coefficient has its zero field value D_o. Hence, assuming the emitter and collector have the same low field mobility,

$$\eta(x_E) = D_o p(x_E) \tag{11a}$$

$$\eta(x_C) = D_o p(x_C) \tag{11b}$$

so that (10) may be rewritten

$$j_c = \frac{qD_o}{\bar{a}}\frac{p(x_E)e^{\psi(x_E)} - p(x_C)e^{\psi(x_C)}}{\displaystyle\int_{x_E}^{x_C} e^{\psi(x)}\,dx}. \tag{12}$$

Since the normalized potentials in (12) are, in general, nonconservative, it is convenient to introduce a conservative electrical potential $\hat{\psi}(x)$ which is everywhere normalized to the lattice temperature T_o. Then in regions where the holes are not heated, their concentration is given by

$$p(x) = n_i e^{\varphi_p(x) - \hat{\psi}(x)}, \tag{13}$$

where n_i is the intrinsic carrier concentration and $\varphi_p(x)$ is the hole

quasi-Fermi level normalized to T_o. Equation (13) can be invoked at x_E and x_C, yielding

$$j_c = \frac{qn_iD_o}{\bar{a}} \cdot \frac{e^{\psi(x_E)-\hat{\psi}(x_E)} \cdot e^{\varphi_p(x_E)} - e^{\psi(x_C)-\hat{\psi}(x_C)} \cdot e^{\varphi_p(x_C)}}{\int_{x_E}^{x_C} e^{\psi(x)-\hat{\psi}(x)} \cdot e^{\hat{\psi}(x)}\, dx}. \tag{14}$$

The relationship between $\psi(x)$ and $\hat{\psi}(x)$ is arbitrary to within a constant, permitting a choice of the coordinate at which $\psi(x)$ and $\hat{\psi}(x)$ coincide. Although (14) is implicitly "gauge invariant," its explicit form will depend on the choice made. The most symmetrical appearance is obtained if one relates $\hat{\psi}(x)$ to $\psi(x)$ by

$$\hat{\psi}(x) = -\frac{q}{kT_o} \int_{x_B}^{x} E\, dx + \psi(x_B), \tag{15}$$

where x_B is any coordinate in the base. Then (14) becomes

$$j_c = \frac{qn_iD_o}{\bar{a}} \frac{\gamma(x_E)e^{\varphi_p(x_E)} - \gamma(x_C)e^{\varphi_p(x_C)}}{\int_{x_E}^{x_C} \gamma(x)e^{\hat{\psi}(x)}\, dx}, \tag{16}$$

where

$$\gamma(x) \equiv e^{\psi(x)-\hat{\psi}(x)} = \exp \int_{x_B}^{x} \frac{qE(T_d - T_o)}{kT_dT_o}\, dx. \tag{17}$$

The function $\gamma(x)$ provides a uniform treatment of the hot carrier effects in (16), which all arise when hole current is drifted in the direction of the field, and power absorption from the field raises the hole diffusion temperature T_d above T_o.

The ICM relation follows from (16) if the quasi-Fermi level for the electrons in the base may be regarded as constant. This implies the absence of substantial dc base majority carrier current, such as would arise if there were both high-level injection and poor current gain.[4] For a constant electron quasi-Fermi level φ_{nb} one has

$$n(x) = n_ie^{\hat{\psi}(x)-\varphi_{nb}} \tag{18}$$

and

$$V_{eb} = \frac{kT_o}{q}(\varphi_p(x_E) - \varphi_{nb}), \tag{19a}$$

$$V_{cb} = \frac{kT_o}{q}(\varphi_p(x_C) - \varphi_{nb}), \tag{19b}$$

where V_{eb} and V_{cb} are respectively the applied emitter-base and collector-base voltages exclusive of ohmic drops. Insertion of (18) and (19) into (16) results in

$$j_c = \frac{n_i^2 q D_a}{\bar{a}} \frac{\gamma(x_E) e^{qV_{eb}/kT_o} - \gamma(x_C) e^{qV_{cb}/kT_o}}{\int_{x_E}^{x_C} \gamma(x) n(x)\, dx}. \tag{20}$$

Letting A denote the active cross-sectional area of the transistor, and defining an effective base majority carrier charge by

$$Q_b^* = qA \int_{x_E}^{x_C} \gamma(x) n(x)\, dx, \tag{21}$$

one arrives at the ICM relation for the collector current I_c.

$$I_c = -\frac{(qn_i A)^2 D_a}{\bar{a}} \frac{\gamma(x_E) e^{qV_{eb}/kT_o} - \gamma(x_C) e^{qV_{cb}/kT_o}}{Q_b^*}. \tag{22}$$

If one neglects carrier heating, $\gamma(x) = 1$ for all x and Q_b^* reduces to Q_b, the total majority charge (within the active region) that communicates with the base terminal. Equation (22) then becomes identical to the integral charge-control relation derived by Gummel.[1]

III. DISCUSSION AND CONCLUSION

We have shown that inclusion of the diffusion coefficient within the gradient operation in the current transport equation automatically eliminates additive contributions to the defining integral for the base charge in the ICM. However, careful examination of the nonconservative potentials appearing in the formalism discloses other hot carrier contributions that have not been previously considered. In equation (22) these are embodied in $\gamma(x_E)$, $\gamma(x_C)$, and Q_b^*. For forward operation of the transistor, $\gamma(x_E) = 1$ because the holes do not absorb power from the emitter junction field. On the other hand, carrier heating can occur in the collector junction and, in accordance with (17), result in $\gamma(x_C) > 1$. However, this effect would be discernible only for reasonably large values of exp (qV_{cb}/kT), requiring the transistor to be in or near saturation. Heating must then be produced by the built-in field. Similar considerations apply to the effective charge defined by (21). Reverse bias of the collector causes the Boltzmann tail of $n(x)$ to fall off very fast within the collector junction and make little contribution to the integral. Saturated or near-saturated operation of the transistor is therefore required for carrier heating to affect Q_b^*. Further-

more, the number of impurities per unit area of the base must be low for the Boltzmann tails within the junctions to make any significant contributions to the total base charge. Since $\gamma(x) > 1$ within the collector junction, Q_b^* will exceed Q_b. Therefore, by increasing $\gamma(x_C)$ and the effective base charge, carrier heating in the collector junction tends to decrease the collector current for a given set of applied voltages. The diminution in I is plausible in view of the decreased effectiveness of the collector junction as a sink for the minority holes diffusing across the base when their mobility within the junction is lowered by carrier heating.

A number of important effects, such as impact ionization and base crowding, have not been included in this treatment. High current gain has been assumed. The question of the ultimate validity of the macroscopic transport equation in inhomogeneous high-field regions has not been addressed.

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4. Gradients in the quasi-Fermi level can also be produced by large transient base charging currents. Ohmic drops across the inactive base can account for quasi-Fermi level gradients therein. In the active base the problem is essentially two-dimensional and beyond the scope of this treatment.